BEFORE THE BOARD OF PUBLIC EDUCATION
OF THE STATE OF MONTANA

In the matter of the adoption of New Rules I through VII, the amendment of ARM 10.53.501, 10.53.502, 10.53.503, 10.53.504, 10.53.505, 10.53.506, 10.53.507, 10.53.508, 10.53.509, 10.53.510, and 10.53.511, and the repeal of ARM 10.53.512, 10.53.513, 10.53.514, 10.53.515, 10.53.516, and 10.53.517 pertaining to Mathematics Content Standards	)))))))))))))))	NOTICE OF PUBLIC HEARING ON PROPOSED ADOPTION, AMENDMENT, AND REPEAL

TO: All Concerned Persons

1. On October 29, 2024, at 9:00 a.m., the Board of Public Education (board) will hold a virtual public hearing to consider the proposed adoption, amendment, and repeal of the above-stated rules. Interested parties may attend the hearing electronically at the following zoom link: https://mt-gov.zoom.us/j/87891928135.

2. The board will make reasonable accommodations for persons with disabilities who wish to participate in this rulemaking process or need an alternative accessible format of this notice. If you require an accommodation, contact the board no later than 5:00 p.m. on October 18, 2024, to advise us of the nature of the accommodation that you need. Please contact McCall Flynn, Executive Director, 46 N. Last Chance Gulch, Suite 2B, P.O. Box 200601, Helena, MT 59620-0601; telephone (406) 444-0302; or email bpe@mt.gov.

3. The rules proposed to be adopted are as follows:

NEW RULE I CORE NUMERIC REASONING STANDARDS (1) The real number system content standards for high school are to:

(a) use reasoning to establish properties of integer exponents, including scientific notation;

(b) represent and perform operations within very large and very small numbers using scientific notation; and

(c) define, manipulate, interpret, and compare real numbers presented through different representations, including both rational and irrational numbers and apply comparisons in context. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities.

AUTH: Mont. Const. Art. X, sec. 9, 20-2-114, 20-7-101, MCA
IMP: Mont. Const. Art. X, sec. 9, 20-2-121, 20-3-106, 20-7-101, MCA

NEW RULE II CORE ALGEBRAIC AND FUNCTIONAL REASONING STANDARDS (1) The functions content standards for high school are to:

(a) interpret parts of an expression, such as terms, factors, and coefficients;

(b) understand the definition of a function and distinguish between functions and relations;

(c) represent functions using tables, graphs with appropriate scales and labels, equations, and verbal situations, while using technology strategically, by:

(i) understanding that different representations highlight different aspects of functions, and choosing the representation that is appropriate for the context; and

(ii) comparing properties of two functions, including when each is represented in a different way;

(d) use function notation, evaluate functions, and interpret statements that use function notation in context. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities;

(e) identify the domain and range of a function, including considering the constraints imposed by context. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities;

(f) understand that a graph of an equation in two variables is the set of all of its solutions plotted in a coordinate plane;

(g) understand that expressions can be rewritten in equivalent forms to make different characteristics or features visible; and

(h) rearrange literal equations to highlight quantities of interest.

(2) The linear functions content standards for high school are to:

(a) understand that linear functions have a constant rate of change;

(b) understand slope as a rate of change and y-intercept as initial value;

(c) represent linear functions using tables, graphs, equations, and verbal situations, while using technology strategically. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities by:

(i) identifying the rate of change and initial value in each representation;

(ii) converting between representations; and

(iii) writing equations for a line perpendicular or parallel to a given line that passes through a given point;

(d) understand that linear equations can be represented in multiple forms and the specific features of each form by:

(i) choosing the form strategically when writing an equation based on given information and intended use;

(ii) converting between slope-intercept, point-slope, and standard form symbolically;

(iii) understanding the relationship between slope-intercept form, the rate of change, and the initial value;

(iv) understanding the relationship between point-slope form, the rate of change, and a given point; and

(v) understanding the relationship between standard form and the x- and y-intercepts;

(e) understand that a solution to a system of equations is a coordinate pair that makes both equations true; and

(f) solve systems of linear equations by graphing, substitution, and elimination, including systems with zero, one, or infinite solutions, while using technology and representations strategically.

(3) The quadratic functions and expressions content standards for high school are to:

(a) understand that quadratic functions do not have a constant rate of change but have a constant second difference over equal intervals and identify the constant second difference in tables;

(b) represent quadratic functions using tables, graphs, equations, and verbal situations, while using technology strategically. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities;

(c) understand that quadratic expressions can be represented in multiple forms and the specific features of each form by:

(i) choosing the form strategically when writing an expression based on given information and intended use;

(ii) converting between factored, standard, and vertex form symbolically and using representations;

(iii) understanding the relationship between factored form and the zeros of the function; and

(iv) understanding the relationship between vertex form and the vertex of the function.

(4) The exponential functions and expressions content standards for high school are to:

(a) understand that exponential functions have a constant common ratio over equal intervals, and identify the common ratio in tables and equations;

(b) understand a as the initial value and b as the growth/decay factor for an exponential function written in standard form, y=a*b^x;

(c) understand the relationship between growth/decay factor and growth/decay rate;

(d) represent exponential functions using tables, graphs, equations, and verbal situations; using technology strategically. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities; and

(e) solve exponential equations graphically, while using technology strategically.

(5) The modeling with functions content standards for high school are to:

(a) model situations in context, with linear, quadratic, and exponential functions. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities by:

(i) determining if a set of data is best modeled by a linear function, quadratic function, exponential function, or none, and explaining why; and

(ii) understanding that there are contexts where solutions may not lie on the curve;

(b) interpret the coefficients in a linear, quadratic, and exponential model in context. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities;

(c) choose and interpret measurement units in formulas, graphs, and data displays to understand problems and to guide problem-solving in modeling situations. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities; and

(d) choose a level of accuracy appropriate to limitations on measurement when reporting quantities in modeling situations. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities.

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NEW RULE III CORE DATA REASONING AND PROBABILITY STANDARDS (1) The quantitative literacy content standards for high school are to:

(a) distinguish between quantitative and categorical data and use representations and analysis techniques that are appropriate for each type;

(b) ask a statistical question to determine whether there appears to be an association between two variables, design and carry out an investigation, and write a persuasive argument based on the results of the investigation; and

(c) distinguish between association and causation.

(2) The visualizing, summarizing, and interpreting data content standards for high school are to:

(a) use technology to organize data, including very large data sets, into a useful and manageable structure;

(b) represent the distribution of univariate quantitative data with plots on the real number line, choosing a format most appropriate to the data set, and representing the distribution of bivariate quantitative data with a scatter plot;

(c) understand that standard deviation measures the variability of a data distribution, and calculate standard deviation using technology;

(d) interpret differences in the shape, center, and spread of quantitative data distributions, in context, accounting for possible effects of outliers on measures of central tendency and variability;

(e) compare and contrast two or more quantitative data distributions, using shape, center, and spread in context. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities;

(f) analyze the relationship between two quantitative data distributions in context that have a linear association. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities by:

(i) using technology strategically, represent two quantitative data distributions on scatter plots;

(ii) describing verbally how the variables are related;

(iii) using technology to find the least-squares regression line (line of best) fit for two quantitative variables;

(iv) understanding that the line of best fit minimizes the square of the residuals; and

(v) understanding correlation as a measure of linear association and using technology, compute the correlation coefficient of a linear relationship;

(g) analyze the relationship between two categorical variables in context. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities by:

(i) summarizing categorical data for two categories in two-way frequency tables and visual representations;

(ii) interpreting relative frequencies for categorical data in context; and

(iii) identifying possible associations and trends in categorical data.

(3) The probability content standards for high school are to:

(a) understand the concept of a sample space and describe events as subsets of a sample space; and

(b) understand the concepts of conditional probability and independence in context. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities by:

(i) determining whether two events, A and B, are independent by using two-way tables, tree diagrams, and/or Venn diagrams, and interpreting the answer in context; and

(ii) computing the conditional probability of event A given event B by using two-way tables, tree diagrams, and/or Venn diagrams, and interpreting the answer in context.

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NEW RULE IV CORE GEOMETRIC REASONING STANDARDS

(1) The transformations content standards for high school are to:

(a) represent transformations in the plane using a variety of methods;

(b) define the congruence of two and show that two figures are congruent by finding a sequence of rigid motions that maps one figure to the other by:

(i) using the definition of congruence in terms of rigid motions to show that two triangles are congruent if, and only if, corresponding pairs of sides and corresponding pairs of angles are congruent; and

(ii) verifying that two triangles are congruent if, but not only if, the following groups of corresponding parts are congruent: angle-side-angle (ASA), side-angle-side (SAS), and side-side-side (SSS);

(c) define the similarity of two figures in terms of similarity transformations by:

(i) verifying that two triangles are similar if, and only if, corresponding pairs of sides are proportional and corresponding pairs of angles are congruent; and

(ii) using the properties of similarity transformations to establish the angle-angle (AA) criterion for two triangles to be similar.

(2) The geometric arguments, reasoning, and proof content standards for high school are to:

(a) investigate, conjecture, prove theorems, and communicate the proofs in a variety of ways by:

(i) proving theorems about lines and angles; theorems include: vertical angles are congruent; when a transversal crosses parallel lines alternate interior angles are congruent and corresponding angles are congruent; and the points on the perpendicular bisector of a line segment are those equidistant from the segment's endpoints;

(ii) proving theorems about triangles; theorems include: the sum of the measures of the interior angles of a triangle is 180˚; the Pythagorean Theorem; the base angles of isosceles triangles are congruent; and a line parallel to one side of a triangle divides the other two sides proportionally;

(iii) proving theorems about parallelograms and other quadrilaterals; theorems include: necessary and sufficient conditions for rectangles, parallelograms, rhombi, and kites; and

(iv) proving theorems about circles; theorems include: the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; and the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

(3) The measurement, problem solving, and geometric modeling content standards for high school are to:

(a) use the Pythagorean Theorem to calculate distance in the coordinate plane;

(b) derive the equation of a circle of a given center and radius using the Pythagorean Theorem;

(c) use similarity to explore and define the sine ratio, cosine ratio, and tangent ratio in terms of right triangles by:

(i) deriving and applying the trigonometric ratios in special right triangles; and

(ii) using trigonometric ratios and the Pythagorean Theorem to solve right triangles;

(d) use geometric shapes, their measures, and their properties to model objects and use those models to solve problems in context. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities by:

(i) modeling and solving problems with 2D shapes by using the perimeter and area of polygons, circles, and composite shapes with portions removed;

(ii) modeling and solving problems with 3D solids by using surface area and volume of solids, including composite solids and solids with portions removed; and

(iii) deriving and applying the relationships between the lengths, perimeters, areas, and volumes of similar figures in relation to their scale factor.

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NEW RULE V CORE PLUS NUMBER AND QUANTITY STANDARDS

(1) The numeric reasoning content standards for high school are to:

(a) extend the properties of exponents to rational exponents, including converting between exponential and radical form; and

(b) understand there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b as real numbers by:

(i) adding, subtracting, multiplying, and dividing complex numbers; and

(ii) finding the conjugate of a complex number.

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NEW RULE VI CORE PLUS ALGEBRAIC AND FUNCTIONAL REASONING STANDARDS (1) The functions, expressions, and inequalities content standards for high school are to:

(a) identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(k x), and f(x + k) for specific values of k (both positive and negative); and

(b) understand the relationship between a function and its inverse.

(2) The polynomial functions content standards for high school are to:

(a) understand polynomials are created by multiplying linear factors;

(b) understand that polynomial expressions can be represented in both factored and standard form, and the specific features of each form by:

(i) choosing the form strategically based on given information and intended use when writing an expression;

(ii) converting between factored and standard form symbolically and using representations (e.g., area model); and

(iii) interpreting the relationship between the factored form of the expression and the zeros of the function;

(c) graph polynomial functions with and without the use of technology, by identifying zeros, relative maxima and minima, and end behavior; and

(d) solve quadratic equations that have complex solutions, and understand why the solutions form a conjugate pair.

(3) The exponential and logarithmic functions content standards for high school are to:

(a) understand logarithmic functions as the inverse of exponential functions;

(b) understand why e is defined as the natural base;

(c) understand that exponential and logarithmic functions can be represented using multiple forms by:

(i) expressing exponential functions in the form f(x)=ab^x and f(x)=Pe^(rt); and

(ii) expressing logarithmic functions in base 10 and base e;

(d) graph logarithmic and exponential functions with and without the use of technology by identifying intercepts, asymptotes, and end behavior; and

(e) solve exponential and logarithmic equations using inverse operations with and without the use of technology.

(4) The trigonometric functions content standards for high school are to:

(a) understand how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers by:

(i) defining the sine and cosine functions in terms of the unit circle; and

(ii) defining the tangent, cotangent, secant, and cosecant functions in terms of sine and cosine;

(b) understand and use the radian measure of an angle, and convert between degree and radian measures;

(c) graph trigonometric functions with and without the use of technology by:

(i) graphing sine and cosine functions, identifying period, midline, and amplitude; and

(ii) graphing tangent functions, identifying period and asymptotes;

(d) solve trigonometric equations with and without the use of technology; and

(e) apply the Law of Sines and the Law of Cosines to find unknown measurements in non-right triangles.

(5) The modeling content standards for high school are to:

(a) model situations in context with polynomial, exponential, logarithmic, and trigonometric functions. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities by:

(i) determining if a set of data is best modeled by a polynomial, exponential, logarithmic, or trigonometric function or none, and explaining why; and

(ii) understanding that there are contexts where solutions may not lie on the curve;

(b) interpret the coefficients in a polynomial, exponential, logarithmic, and trigonometric model in context. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities;

(c) use and interpret units correctly in modeling situations; and

(d) choose a level of accuracy appropriate to limitations on measurement when reporting quantities in modeling situations.

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NEW RULE VII CORE PLUS DATA AND REASONING STANDARDS

(1) The normal distribution content standards for high school are to:

(a) use technology to find the mean and standard deviation of a normally distributed data set and apply the empirical rule to estimate population percentages; and

(b) estimate areas under a normal curve to solve problems in context, using calculators, spreadsheets, and tables as appropriate.

(2) The experimental design content standards for high school are to:

(a) describe the purposes of and differences among sample surveys, experiments, and observational studies and explain how randomization relates to each;

(b) describe differences between randomly selecting samples and randomly assigning subjects to experimental treatment groups in terms of inferences drawn regarding a population versus regarding cause and effect by:

(i) explaining the consequences, due to uncontrolled variables, of non-randomized assignment of subjects to groups in experiments; and

(ii) evaluating where bias, including sampling, response, or nonresponse bias, may occur in surveys, and whether results are representative of the population of interest;

(c) evaluate the effect of sample size on the expected variability in the sampling distribution of a sample statistic by:

(i) simulating a sampling distribution of sample means from a population with a known distribution, observing the effect of the sample size on the variability; and

(ii) demonstrating that the standard deviation of each simulated sampling distribution is the known standard deviation of the population divided by the square root of the sample size.

(3) The statistical inference using simulation content standards for high school are to:

(a) distinguish between a statistic and a parameter and use statistical processes to make inferences about population parameters based on statistics from random samples from that population;

(b) estimate a population parameter from a representative sample by:

(i) understanding why the sample statistic is the best estimate for the associated population parameter;

(ii) understanding that sampling variability introduces uncertainty in the estimate, and account for the uncertainty with a confidence interval by:

(A) using resampling with replacement from an observed sample to produce a sampling distribution;

(B) verifying that a sampling distribution is centered at the population mean and approximately normal if the sample size is large enough;

(C) verifying that 95% of sample means are within two standard deviations of the sampling distribution from the population mean; and

(D) creating and interpreting a 95% confidence interval based on an observed mean from a sampling distribution;

(c) use data from a randomized experiment to test the hypothesis that two groups are equal by:

(i) interpreting the difference or ratio between the group means as the observed effect between the groups; and

(ii) understanding that an observed effect may be due to randomization and using a randomization test (repeatedly reshuffling the observed data into new groups) to determine the probability that an observed effect is due to randomization alone.

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4. The rules proposed to be amended provide as follows, new matter underlined, deleted matter interlined:

10.53.501 STANDARDS FOR MATHEMATICAL PRACTICE FOR GRADES K-12 (1) Mathematical practice standard 1 is to ~~make sense of problems~~ problem solve and persevere ~~in solving them~~. Mathematically proficient students:

(a) ~~explain the meaning of a problem and restate it in their words~~ make conjectures, plan, and follow solution strategies;

(b) ~~analyze given information to develop possible strategies for solving the problem~~ evaluate their progress and accuracy;

(c) ~~identify and execute appropriate strategies to solve the problem;~~ engage in sense-making and self-monitoring; and

(d) ~~evaluate progress toward the solution and make revisions if necessary; and~~ persevere in seeking solutions, and value alternative approaches.

~~(e) check their answers using a different method and continually ask "Does this make sense?".~~

(2) Mathematical practice standard 2 is to ~~reason~~ abstract~~ly~~ and ~~quantitatively~~ generalize. Mathematically proficient students~~:~~ are able to decontextualize and symbolically represent both mathematical and non-

mathematical situations to search for and analyze regularities, patterns, and structures.

~~(a) make sense of quantities and their relationships in problem situations;~~

~~(b) use varied representations and approaches when solving problems;~~

~~(c) know and flexibly use different properties of operations and objects; and~~

~~(d) change perspectives, generate alternatives, and consider different options.~~

(3) Mathematical practice standard 3 is to ~~construct viable arguments and critique the reasoning of others~~ justify and prove. Mathematically proficient students~~:~~ create, evaluate, justify, and refute mathematical claims in developmentally and mathematically appropriate ways.

~~(a) understand and use prior learning in constructing arguments;~~

~~(b) habitually ask "why" and seek an answer to that question;~~

~~(c) question and problem-pose;~~

~~(d) develop questioning strategies to generate information;~~

~~(e) seek to understand alternative approaches suggested by others and as a result, adopt better approaches;~~

~~(f) justify their conclusions, communicate them to others, and respond to the arguments of others; and~~

~~(g) compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and if there is a flaw in an argument, explain what it is.~~

(4) Mathematical practice standard 4 is to model with mathematics. Mathematically proficient students:

(a) ~~apply the mathematics they know to solve problems arising in everyday life, society, and the workplace~~ make sense of a scenario;

(b) ~~make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later;~~ identify a problem to be solved and mathematize it; and

(c) ~~identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas; and~~ apply a mathematical model to reach a solution and verify its viability.

~~(d) analyze mathematical relationships to draw conclusions.~~

(5) Mathematical practice standard 5 is to ~~use appropriate tools strategically~~ represent. Mathematically proficient students:

(a) ~~use tools when solving a mathematical problem and to deepen their understanding of concepts (e.g., pencil and paper, physical models, geometric construction and measurement devices, graph paper, calculators, computer-based algebra, or geometry systems)~~ recognize, use, create, interpret, and translate representations using appropriate methods and tools; and

(b) ~~make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations and detect possible errors by strategically using estimation and other mathematical knowledge~~ understand multiple ways of representing mathematical ideas and how they are related.

(6) Mathematical practice standard 6 is to ~~attend to precision~~ collaborate mathematically. Mathematically proficient students~~:~~ engage in mathematics as a

social enterprise through discussion and collaborative inquiry where ideas are offered, debated, connected, and built upon toward solutions, shared understanding, and appreciation of other perspectives.

~~(a) communicate their understanding of mathematics to others;~~

~~(b) use clear definitions and state the meaning of the symbols they choose, including using the equal sign consistently and appropriately;~~

~~(c) specify units of measure and use label parts of graphs and charts; and~~

~~(d) strive for accuracy.~~

(7) Mathematical practice standard 7 is to ~~look for and make use of structure~~ culturally connect. Mathematically proficient students:

(a) ~~look for, develop, generalize, and describe a pattern orally, symbolically, graphically, and in written form~~ recognize cultural connections and contributions to mathematics; and

(b) ~~apply and discuss properties~~ appreciate the role of mathematics in various cultural contexts, including those of tribally specific Montana Indigenous Peoples.

~~(8) Mathematical practice standard 8 is to look for and express regularity in repeated reasoning. Mathematically proficient students:~~

~~(a) look for mathematically sound shortcuts; and~~

~~(b) use repeated applications to generalize properties.~~

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10.53.502 MONTANA KINDERGARTEN MATHEMATICS CONTENT STANDARDS (1) Mathematics counting and cardinality standards for kindergarten are to:

(a) flexibly count to 100 by ones and by tens;

(b) count ~~forward~~ beginning from a given number within the known sequence ~~(instead of having to begin at 1)~~;

(c) write numbers from 0-20 and represent a number of objects with a written numeral 0-20 ~~(with 0 representing a count of no objects)~~;

(d) understand the relationship between numbers and quantities and connect counting to cardinality by recognizing that each successive number name refers to a quantity that is one larger within a normal counting sequence;

~~(i) when counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object from a variety of cultural contexts, including those of Montana American Indians;~~

~~(ii) understand that the last number name said tells the number of objects counted and the number of objects is the same regardless of their arrangement or the order in which they were counted;~~

~~(iii) understand that each successive number name refers to a quantity that is one larger;~~

(e) count to answer "how many?" ~~questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration and given a number from 1-20, count out that many objects~~

~~from a variety of cultural contexts, including those of Montana American Indians~~ ~~including those of Montana American Indians~~ in a variety of arrangements and, given a number, produce a set within 20;

(f) identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group~~, e.g., by using matching and counting strategies~~; and

(g) compare two numbers between 1 and 10 presented as written numerals.

(2) Mathematics operations and algebraic thinking content standards for kindergarten are to:

(a) represent addition and subtraction ~~with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations~~ in multiple ways;

(b) solve addition and subtraction ~~word problems from a variety of cultural contexts, including those of Montana American Indians, and add and subtract within 10, e.g., by using objects or drawings to represent the problem~~ problems in context within 10. This standard should incorporate cultural context relating to Montana Indigenous People and local communities;

(c) decompose numbers less than or equal to 10 into pairs in ~~more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1)~~ multiple ways;

(d) for any number from 1 to 9, find the number that makes 10 when added to the given number~~, e.g., by using objects or drawings, and record the answer with a drawing or equation~~; ~~and~~

(e) ~~fluently~~ flexibly, accurately add and subtract within 5~~.~~; and

(f) recognize the characteristics of the commutative property in addition.

(3) Mathematics number and operations in base ten content standard for kindergarten is to:

(a) compose and decompose numbers from 11-19 into ten ones and ~~some~~ further ones, ~~e.g., by using objects or drawings;~~ in multiple ways and record each composition or decomposition by a drawing or an equation ~~(such as 18 = 10 + 8); and understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones~~.

(4) Mathematics measurement and data content standards for kindergarten are to:

(a) describe several measurable attributes of ~~objects, such as length or weight and describe several measurable attributes of~~ a single object;

(b) directly compare two objects with a measurable attribute in common~~, to see which object has "more of"/"less of" the attribute and describe the difference; for example, directly compare the heights of two children and describe one child as taller/shorter; and~~ using comparative language;

(c) classify, count, and sort objects ~~from a variety of cultural contexts, including those of Montana American Indians,~~ into ~~given~~ categories~~, count the numbers of objects in each category, and sort the categories by count~~. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities;

(d) describe attributes and identify the names of coins; and

(e) explain time in days, months, years, and seasons.

(5) Mathematics geometry content standards for kindergarten are to:

(a) describe the relative positions of objects, ~~including those of Montana American Indians,~~ in ~~the~~ their environment ~~using names of shapes and describe the relative positions of these objects using terms such as: above, below, beside, in front of, behind, and next to~~. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities;

(b) correctly name shapes regardless of their orientations or overall size;

(c) identify shapes as two-dimensional ~~(lying in a plane, "flat")~~ or three-dimensional ~~("solid")~~;

(d) analyze and compare two- and three-dimensional shapes~~, in different sizes and orientations,~~ using informal language ~~to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners"),~~ and other attributes ~~(e.g., having sides of equal length)~~;

(e) model shapes in the ~~world from a variety of cultural contexts, including those of Montana American Indians, by building shapes from components (e.g., sticks and clay balls) and drawing shapes~~ environment. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities; and

(f) compose simple shapes to form larger shapes~~; for example, "Can you join these two triangles with full sides touching to make a rectangle?"~~.

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10.53.503 MONTANA GRADE 1 MATHEMATICS CONTENT STANDARDS

(1) Mathematics operations and algebraic thinking content standards for Grade 1 are to:

(a) use addition and subtraction within 20 to solve ~~word~~ problems ~~within a cultural context, including those of Montana American Indians, involving situations of adding to, taking from, putting together, taking apart, and comparing with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem~~ of all types. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities;

(b) solve ~~word~~ problems ~~within a cultural~~ in context, ~~including those of Montana American Indians,~~ that call for addition of three whole numbers ~~whose~~ with a sum ~~is~~ less than or equal to 20~~, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem~~ in context of all types. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities;

(c) ~~apply properties of operations as strategies~~ flexibly compose and decompose numbers to add and subtract; ~~for example: if 8 + 3 = 11 is known, then 3 + 8 = 11 is also known (commutative property of addition); to add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12 (associative property of addition);~~

(d) understand subtraction as an unknown-addend problem; ~~for example, subtract 10 - 8 by finding the number that makes 10 when added to 8;~~

(e) relate counting to addition and subtraction ~~(e.g., by counting on 2 to add 2)~~;

(f) flexibly, accurately, and efficiently add and subtract within ~~20 demonstrating fluency for addition and subtraction within~~ 10; ~~use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13);~~

(g) use multiple strategies to add and subtract within 20;

~~(g)~~(h) understand the meaning of the equal sign and determine if equations ~~involving addition and subtraction~~ are true or false; ~~for example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2;~~ and

~~(h)~~(i) determine the unknown ~~whole~~ number in an addition or subtraction equation relating to three ~~whole~~ numbers~~; for example, determine the unknown number that makes the equation true in each of the equations: 8 + ? = 11, 5 = ? - 3, 6 + 6 = ?~~.

(2) Mathematics number and operations in base ten content standards for Grade 1 are to:

(a) ~~count to 120, starting at any number less than 120 and read and write numerals and represent a number of objects with a written numeral in this range;~~ flexibly count, read, write, and represent numbers to 120;

(b) understand that ~~the two digits of a two-digit number represent amounts of tens and ones and understand the following as special cases~~ ten is a unit composed of ten ones and that a two-digit number represents tens and ones;

~~(i) 10 can be thought of as a bundle of ten ones called a "ten";~~

~~(ii) the numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones;~~

~~(iii) the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones);~~

(c) compare two two-digit numbers ~~based on meanings of the tens and ones digits, recording the results of comparisons with the~~ using comparison symbols >, =, and <;

(d) ~~add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used; understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten;~~ build a foundation for addition within 100 by:

(i) adding two-digit to one-digit numbers; and

(ii) adding multiples of 10 to two-digit numbers;

(e) using place value, given a two-digit number, ~~mentally~~ find 10 more or 10 less than the number~~, without having to count; explain the reasoning used~~; and

(f) subtract multiples of 10 ~~in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences) using concrete models or drawings and~~

~~strategies based on place value, properties of operations, and/or the relationship between addition and subtraction, relate the strategy to a written method, and explain the reasoning used.~~ from a two-digit number.

(3) Mathematics measurement and data content standards for Grade 1 are to:

(a) order three objects ~~from a variety of cultural contexts, including those of Montana American Indians,~~ by length and compare the lengths of two objects indirectly by using a third object. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities;

(b) express the length of an object as a whole number of length units~~, by laying multiple copies of a shorter object (the length unit) end to end~~; understand that the ~~length~~ measurement of an object is the number of same-size length units that span it with no gaps or overlaps ~~and limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps~~;

(c) tell and write time in hours and half-hours using analog and digital clocks; ~~and~~

(d) identify the value of coins; and

~~(d)~~(e) organize, represent, and interpret data with up to three categories ~~and ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.~~ by:

(i) asking and answering questions about the total number of data points;

(ii) identifying how many are in each category; and

(iii) analyzing differences between categories.

(4) Mathematics geometry content standards for Grade 1 are to:

(a) distinguish between defining attributes ~~(e.g., triangles are closed and three-sided)~~ versus nondefining attributes ~~(e.g., color, orientation, overall size) and build and draw shapes to possess defining attributes~~;

(b) build and draw shapes to possess defining attributes;

~~(b)~~(c) compose ~~two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape~~ new shapes using two- and three-dimensional shapes; and

~~(c)~~(d) partition circles and rectangles into two and four equal shares; describe the shares using the words: halves, fourths, and quarters~~, and use the phrases half of, fourth of, and quarter of; describe the whole as two of, or four of the shares; and understand for these examples that decomposing into more equal shares creates smaller shares~~.

AUTH: Mont. Const. Art. X, sec. 9, 20-2-114, 20-7-101, MCA
IMP: Mont. Const. Art. X, sec. 9, 20-2-121, 20-3-106, 20-7-101, MCA

10.53.504 MONTANA GRADE 2 MATHEMATICS CONTENT STANDARDS

(1) Mathematics operations and algebraic thinking content standards for Grade 2 are to:

(a) use addition and subtraction within 100 to solve one- and two-step ~~word~~ problems ~~involving situations within a cultural~~ in context~~, including those of Montana~~

~~American Indians, of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem;~~ involving all problem types. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities;

(b) ~~fluently~~ flexibly, accurately, and efficiently add and subtract within 20 using mental strategies ~~and by the end of Grade 2, know from memory all sums of two one-digit numbers~~;

(c) determine whether a group of objects, ~~(~~up to 20~~)~~, has an odd or even number of members~~, e.g., by pairing objects or counting them by 2s and write an equation to express an even number as a sum of two equal addends~~; and

(d) use addition to find the total number of objects arranged in rectangular arrays ~~with up to five rows and up to five columns and write an equation to express the total as a sum of equal addends~~.

(2) Mathematics number and operations in base ten content standards for Grade 2 are to:

(a) understand one hundred is a unit composed of ten tens and that ~~the three digits of a~~ three-digit ~~number~~ numbers represent amounts of hundreds, tens, and ones~~, e.g., 706 equals 7 hundreds, 0 tens, and 6 ones and understand the following special cases:~~;

~~(i) 100 can be thought of a s a bundle of ten tens — called a "hundred;" and~~

~~(ii) the numbers 100, 200, 300, 400, 500, 600, 700, 800, and 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones);~~

(b) ~~count within 1000;~~ skip-count by 5s, 10s, and 100s;

(c) ~~read and write~~ flexibly count, read, write, and represent numbers to 1000 ~~using base-ten numerals, number names, and expanded form~~;

(d) compare two three-digit numbers ~~based on meanings of the hundreds, tens, and ones digits,~~ using >, =, and < symbols ~~to record the results of comparisons~~;

(e) ~~fluently~~ flexibly, accurately, and efficiently add and subtract within 100 using multiple strategies ~~based on place value, properties of operations, and/or the relationship between addition and subtraction~~;

(f) add up to four two-digit numbers using multiple strategies ~~based on place value and properties of operations~~;

(g) add and subtract within 1000 using ~~concrete models or drawings and~~ multiple strategies ~~based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method; understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones, and sometimes it is necessary to compose or decompose tens or hundreds~~;

(h) ~~mentally add 10 or 100 to a given number 100-900 and mentally subtract 10 or 100 from a given number 100-900~~ using place value, add or subtract 10 or 100 from a given number; and

(i) ~~explain why addition and subtraction strategies work using place value and the properties of operations.~~ understand and make connections between different strategies for addition and subtraction.

(3) Mathematics measurement and data content standards for Grade 2 are to:

(a) measure the length of an object by selecting and using appropriate tools ~~such as rulers, yardsticks, meter sticks, and measuring tapes~~;

(b) ~~measure the length of an object twice, using length units of different lengths for the two measurements and describe how the two measurements relate to the size of the unit chosen;~~ understand the relationship between unit sizes and number of units by measuring a single object using two different units of common measurement;

(c) estimate lengths using units of ~~inches, feet, centimeters, and meters~~ common measurement;

(d) measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit;

(e) use addition and subtraction within 100 to solve ~~word~~ problems ~~within a cultural~~ in context~~, including those of Montana American Indians,~~ involving lengths that are given in the same units~~, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem~~. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities;

(f) represent whole numbers as lengths from 0 ~~on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ...~~ and represent ~~whole-number~~ sums and differences within 100 on a number line ~~diagram~~;

(g) tell and write time from analog and digital clocks to the nearest five minutes using a.m. and p.m.;

(h) solve ~~word~~ problems in context involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately; ~~for example: if you have two dimes and three pennies, how many cents do you have?;~~

(i) generate measurement data ~~by measuring lengths of several objects to the nearest whole unit or by making repeated measurements of the same object and show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units;~~ and present the data in multiple ways. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities;

(j) ~~draw a picture graph and a bar graph (with single unit scale) to represent a data set from a variety of cultural contexts, including those of Montana American Indians, with up to four categories and solve simple put together, take apart and compare problems using information presented in a bar graph.~~ organize, represent, and interpret data with up to four categories. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities; and

(k) solve addition and subtraction problems of all types using data presented.

(4) Mathematics geometry content standards for Grade 2 are to:

(a) recognize and draw shapes having specified attributes~~, such as a given number of angles or a given number of equal faces and identify triangles, quadrilaterals, pentagons, hexagons, and cubes~~;

(b) partition a rectangle into rows and columns of same size squares and ~~count to~~ find the total number ~~of them~~; and

(c) partition circles and rectangles into ~~two, three, or four~~ equal shares~~;~~

~~describe the shares using the words halves, thirds, half of, a third of, etc.; describe the whole as two halves, three thirds, four fourths; and~~, recognize that equal shares ~~of identical wholes~~ need not have the same shape, and express the shares in two-halves, three-thirds, and four-fourths.

AUTH: Mont. Const. Art. X, sec. 9, 20-2-114, 20-7-101, MCA
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10.53.505 MONTANA GRADE 3 MATHEMATICS CONTENT STANDARDS

(1) Mathematics operations and algebraic thinking content standards for Grade 3 are to:

(a) ~~interpret~~ understand products of whole numbers~~, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each; for example, describe a context in which a total number of objects can be expressed as 5 × 7;~~ as the total number found by multiplying a number of groups by the number of objects per group;

(b) ~~interpret~~ understand whole-number quotients of whole numbers~~:, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each; for example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8;~~

(i) as the number of objects in each group with the total quantity divided equally into a number of shares; and

(ii) as the number of shares when a total number of objects is partitioned into equal-sized groups;

(c) use multiplication and division within 100 to solve ~~word~~ problems in context in situations involving equal groups, arrays, and measurement quantities~~, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem~~;

(d) determine the unknown whole number in a multiplication or division equation relating three whole numbers; ~~for example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = ? ÷ 3, 6 × 6 = ?;~~

(e) apply ~~properties of operations as strategies to multiply and divide; for example: if 6 × 4 = 24 is known, then 4 × 6 = 24 is also known (commutative property of multiplication); 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30 (associative property of multiplication); knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56 (distributive property)~~ the commutative property of multiplication, associative property of multiplication, and distributive property of multiplication over addition on whole numbers as strategies to multiply;

(f) ~~understand~~ use division as an unknown factor problem; ~~for example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8;~~

(g) ~~fluently~~ flexibly, accurately, and efficiently multiply and divide within 100, using strategies such as the relationship between multiplication and division ~~(e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8)~~ or properties of operations ~~and by the end of Grade 3, know from memory all products of two one-digit numbers~~;

(h) solve two step ~~word~~ problems in context using the four operations ~~within cultural contexts, including those of Montana American Indians~~; represent these problems using equations with a letter standing for the unknown quantity; and assess the reasonableness of answers using mental computation and estimation strategies ~~including rounding~~. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities; and

(i) identify arithmetic patterns ~~(including patterns in the addition table or multiplication table)~~ and explain them using properties of operations~~; for example, observe that four times a number is always even, and explain why four times a number can be decomposed into two equal addends~~.

(2) Mathematics number and operations in base ten content standards for Grade 3 are to:

(a) use place value understanding to round whole numbers to the nearest 10 or 100;

(b) ~~fluently~~ flexibly, accurately, and efficiently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction; and

(c) multiply one-digit whole numbers by multiples of 10 in the range 10-90 ~~(e.g., 9 × 80, 5 × 60)~~ using strategies based on place value and properties of operations.

(3) Mathematics number and operations fractions content standards for Grade 3 are to:

(a) understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts and understand a fraction a/b as the quantity formed by a parts of size 1/b;

(b) understand a fraction as a number on the number line ~~and represent fractions on a number line diagram;~~ by:

(i) ~~represent~~ representing a unit fraction ~~1/b~~ on a number line ~~diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts, recognize that each part has size 1/b, and that the endpoint of the part based at 0 locates the number 1/b on the number line~~; and

(ii) representing a fraction ~~a/b on a number line diagram by marking off a lengths 1/b from 0 and recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line~~ as multiple copies of a unit fraction on a number line; and

(iii) representing fractions on a number line;

(c) ~~explain~~ understand the equivalence of fractions in special cases and compare fractions by reasoning about their size~~;~~ by:

(i) ~~understand~~ understanding two fractions as equivalent ~~(equal)~~ if they are the same size or the same point on a number line;

(ii) ~~recognize~~ recognizing and ~~generate~~ generating simple equivalent fractions~~, e.g., 1/2 = 2/4, 4/6 = 2/3 and explain why the fractions are equivalent, e.g., by using a visual fraction model~~; and by demonstrating or justifying why the fractions are equivalent;

(iii) ~~express~~ writing whole numbers as fractions, ~~and recognize~~ recognizing fractions that are equivalent to whole numbers~~; for example: express 3 in the form 3~~

~~= 3/1; recognize that 6/1 = 6;~~, and ~~locate 4/4 and 1 at the same point of a~~ locating them on the number line ~~diagram~~; ~~and~~

(iv) ~~compare~~ comparing two fractions with the same numerator or the same denominator by reasoning about their size~~; recognize~~ and recognizing that comparisons are valid only when the two fractions refer to the same whole~~; record the results of comparisons with the symbols >, =, or <; and justify the conclusions, e.g., by using a visual fraction model.~~; and

(v) recording the results of fraction comparisons with the symbols >, =, or < and justifying the conclusions.

(4) Mathematics measurement and data content standards for Grade 3 are to:

(a) tell and write time on an analog and digital clock to the nearest minute and measure time intervals in minutes and solve ~~word~~ problems in context involving addition and subtraction of time intervals in minutes~~, e.g., by representing the problem on a number line diagram~~;

(b) measure and estimate liquid volumes and masses of objects using ~~standard units of grams (g), kilograms (kg), and liters (l) and~~ customary and metric units by ~~add~~ adding, ~~subtract~~ subtracting, ~~multiply~~ multiplying, ~~or divide~~ and dividing to solve one-step ~~word~~ problems in context that involve ~~involving~~ masses or volumes that are given in the same units~~, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem~~;

(c) draw a scaled picture graph and a scaled bar graph to represent a data set with several categories~~, within cultural contexts including those of Montana American Indians~~; solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs.~~; for example, draw a bar graph in which each square in the bar graph might represent five pets~~ This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities;

(d) generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch and show the data by making a line plot where the horizontal scale is marked off in appropriate units~~, i.e. whole numbers, halves, or quarters~~;

(e) recognize area as an attribute of plane figures and understand concepts of area measurement~~;~~ by:

(i) understanding that a square with side length 1 unit, called "a unit square," is said to have "one square unit" of area and can be used to measure area; and

(ii) understanding that a plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units;

(f) measure areas by counting unit squares ~~(square cm, square m, square in, square ft, and improvised units)~~;

(g) relate area to the operations of multiplication and addition~~;~~ by:

(i) ~~find~~ finding the area of a rectangle with whole-number side lengths by tiling it, and ~~show~~ showing that the area is the same as would be found by multiplying the side lengths;

(ii) ~~multiply~~ multiplying side lengths to find areas of rectangles with whole-number side lengths ~~in the context of~~ while solving ~~real-world and mathematical~~

problems in context and ~~represent~~ representing whole-number products as rectangular areas ~~in mathematical reasoning~~;

(iii) ~~use~~ using tiling ~~to show in a concrete case that the~~ and area models to represent the distributive property in finding area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a × c ~~and use area models to represent the distributive property in mathematical reasoning~~;

(iv) ~~recognize~~ recognizing area as additive; ~~find~~ finding areas of ~~rectilinear~~ straight-line figures by decomposing them into nonoverlapping rectangles and adding the areas of the nonoverlapping parts.~~; and apply this technique to solve real-world problems, including those of Montana American Indians~~ This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities; and

(h) solve ~~real-world and mathematical~~ problems in context involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

(5) Mathematics geometry content standards for Grade 3 are to:

(a) understand that shapes in different categories ~~(e.g., rhombuses, rectangles, and others)~~ may share attributes ~~(e.g., having four sides)~~ and that the shared attributes can define a larger category ~~(e.g., quadrilaterals)~~; recognize rhombuses, rectangles, and squares as examples of quadrilaterals; and draw examples of quadrilaterals that do not belong to any of these subcategories; and

(b) partition shapes into parts with equal areas; express the area of each part as a unit fraction of the whole~~; for example, partition a shape into four parts with equal area, and describe the area of each part as 1/4 of the area of the shape~~.

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IMP: Mont. Const. Art. X, sec. 9, 20-2-121, 20-3-106, 20-7-101, MCA

10.53.506 MONTANA GRADE 4 MATHEMATICS CONTENT STANDARDS

(1) Mathematics operations and algebraic thinking content standards for Grade 4 are to:

(a) interpret a multiplication equation as a multiplicative comparison, ~~e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5~~ and represent verbal statements of multiplicative comparisons as multiplication equations;

(b) multiply or divide to solve ~~word~~ problems in context that involve ~~involving~~ multiplicative comparison~~, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem,~~ and ~~distinguishing~~ multiplicative comparison from additive comparison;

(c) solve multistep ~~word~~ problems in context ~~within cultural contexts, including those of Montana American Indians,~~ with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted; represent these problems using equations with a letter standing for the unknown quantity; and assess the reasonableness of answers using mental computation and estimation strategies. ~~including rounding~~ This

standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities;

(d) find all factor pairs for a whole number in the range 1-100; recognize that a whole number is a multiple of each of its factors; determine whether a given whole number in the range 1-1000 is a multiple of a given one-digit number; and determine whether a given whole number in the range 1-100 is prime or composite; and

(e) ~~generate~~ analyze a number or shape pattern~~s~~ that follows a given rule; identify ~~apparent~~ and explain informally features of the pattern that were not explicit in the rule itself; ~~for example, given the rule "add 3" and the starting number 1,~~ generate terms in the resulting sequence and observe ~~that the terms appear to alternate between odd and even numbers; explain informally why the numbers will continue to alternate in this way~~ the pattern.

(2) Mathematics number and operations in base ten content standards for Grade 4 are to:

(a) recognize that in a multi-digit whole number, ~~a digit in one~~ each place represents ten times ~~what it represents in~~ the place to its right; ~~for example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division;~~

(b) read and write multi-digit whole numbers using ~~base ten numerals, number names,~~ standard form, word form, and expanded form and compare two multi-digit numbers based on ~~meanings~~ the value of the digits in each place, using >, =, and < symbols ~~to record the results of comparisons~~;

(c) use place value understanding to round multi-digit whole numbers to any place;

(d) ~~fluently~~ accurately and efficiently add and subtract multi-digit whole numbers using the standard algorithm;

(e) multiply a whole number of up to four digits by a one-digit whole number; multiply two two-digit numbers, flexibly using strategies based on place value and the properties of operations; and illustrate and explain the calculation by using equations, rectangular arrays, and/or area models; and

(f) find whole number quotients and remainders with up to four-digit dividends and one-digit divisors, flexibly using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division and illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

(3) Mathematics number and operations ~~-~~ in fractions content standards for Grade 4 are to:

(a) explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models ~~with attention to how the number and size of the parts differ even though the two fractions themselves are the same size~~ and use this principle to recognize and generate equivalent fractions;

(b) compare two fractions with different numerators and different denominators, ~~e.g.,~~ by creating common denominators or numerators, or by comparing to a benchmark fraction ~~such as 1/2~~; recognize that comparisons are valid only when the two fractions refer to the same whole; record the results of comparisons with symbols >, =, or <; and justify the conclusions~~, e.g., by using a visual fraction model~~;

(c) understand a fraction a/b with a > 1 as a sum of fractions 1/b~~;~~ by:

(i) ~~understand~~ understanding addition and subtraction of fractions as joining and separating parts referring to the same whole;

(ii) ~~decompose~~ decomposing a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation; ~~justify decompositions, e.g., by using a visual fraction model; for example: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8~~;

(iii) ~~add~~ adding and ~~subtract~~ subtracting mixed numbers with like denominators, ~~e.g.,~~ by replacing each mixed number with an equivalent improper fraction, ~~and/or by using properties of operations and the relationship between addition and subtraction~~ or other efficient strategies; and

(iv) ~~solve~~ solving ~~word~~ problems ~~within cultural contexts, including those of Montana American Indians,~~ in context that involve ~~involving~~ addition and subtraction of fractions referring to the same whole and having like denominators., ~~e.g., by using visual fraction models and equations to represent the problem~~ This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities;

(d) apply and extend previous understandings of multiplication to multiply a fraction by a whole number~~;~~ by:

(i) ~~understand~~ understanding a fraction a/b as a multiple of 1/b~~; for example, use a visual fraction model to represent 5/4 as the product 5 × (1/4)~~, and recording the conclusion by the equation ~~5/4 = 5 × (1/4)~~ a/b = a*(1/b);

(ii) ~~understand~~ understanding a multiple of a/b as a multiple of 1/b, ~~and use~~ using this ~~understanding~~ to multiply a fraction by a whole number~~; for example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5)~~, and recognizing ~~this product as 6/5 (in general,~~ n × (a/b) = (n × a)/b~~)~~; and

(iii) ~~solve word~~ solving problems ~~within cultural contexts, including those of Montana American Indians,~~ in context involving multiplication of a fraction by a whole number., ~~e.g., by using visual fraction models and equations to represent the problem; for example, if each person at a party will eat 3/8 of a pound of roast beef and there will be five people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie? As a contemporary American Indian example, for family/cultural gatherings, the Canadian and Montana Cree bake bannock made from flour, salt, grease, and baking soda, in addition to 3/4 cup water per pan. When making four pans, how much water will be needed?~~ This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities;

(e) express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100; ~~for example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100;~~

(f) use decimal notation for fractions with denominators 10 or 100; ~~for example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; and locate 0.62 on a number line diagram;~~ and

(g) compare two decimals to hundredths by reasoning about their size; recognize that comparisons are valid only when the two decimals refer to the same whole; record the results of comparisons with the symbols >, =, or <; and justify the conclusions~~, e.g., by using a visual model~~.

(4) Mathematics measurement and data content standards for Grade 4 are to:

(a) know relative sizes of ~~measurement~~ units within one system of ~~units including km, m, cm, kg, g, lb., oz., l, ml, hr, min., and sec.; within a single system of~~ measurement~~,~~ and within the system, express measurements ~~in~~ of a larger unit in terms of a smaller unit; ~~record measurement equivalents in a two-column table; for example know that 1 ft is 12 times as long as 1 in.; express the length of a four ft snake as 48 in.; generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...;~~

(b) use the four operations to solve ~~word~~ problems ~~within cultural contexts, including those of Montana American Indians, involving~~ in context using distances, intervals of time, liquid volumes, masses of objects, and money~~;~~, including problems ~~involving~~ with simple fractions or decimals and problems that require expressing measurements given in a larger unit in terms of a smaller unit, represent measurement quantities using diagrams ~~such as number line diagrams~~ that feature a measurement scale. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities;

(c) apply the area and perimeter formulas for rectangles ~~in real-world and mathematical problems; for example, find the width of a rectangular room given the area of the flooring and the length by viewing the area formula as a multiplication equation with an unknown factor~~ including problems in context;

(d) make a line plot to display a data set of measurements in fractions of a unit ~~(1/2, 1/4, 1/8);~~ and solve problems involving addition and subtraction of fractions by using information presented in line plots; ~~for example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect or arrow/spearhead collection;~~

(e) recognize angles as geometric shapes that are formed wherever two rays share a common endpoint and understand concepts of angle measurement by:

(i) ~~an angle is measured with reference to a circle with its center at the common endpoint of the rays; by considering the fraction of the circular arc between the points where the two rays intersect the circle, an angle that turns through 1/360 of a circle is called a "one-degree angle" and can be used to measure angles~~ understanding that an angle is formed by two rays with a common endpoint at the center of a circle that measures a total of 360 degrees, and a single-degree unit measure is equal to 1/360th of the circle; and

(ii) understanding that an angle that turns through n one-degree angles is said to have an angle measure of n degrees;

(f) measure angles in whole-number degrees using a protractor and sketch angles of specified measure; and

(g) recognize angle measure as additive; when an angle is decomposed into nonoverlapping parts, the angle measure of the whole is the sum of the angle measurers of the parts; solve addition and subtraction problems to find unknown angles on a diagram ~~in real-world and mathematical~~ including problems in context~~; e.g., by using an equation with a symbol for the unknown angle measure~~.

(5) Mathematics geometry content standards for Grade 4 are to:

(a) draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines and identify these in two-dimensional figures;

(b) classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size; recognize right triangles as a category; and identify right triangles; and

(c) recognize a line of symmetry for a two-dimensional figure~~, including those found in Montana American Indian designs,~~ as a line across the figure such that the figure can be folded along the line into matching parts~~;~~, identify line-symmetric figures~~;~~, and draw lines of symmetry. This standard should incorporate designs and cultural context relating to Montana Indigenous Peoples and local communities.

AUTH: Mont. Const. Art. X, sec. 9, 20-2-114, 20-7-101, MCA
IMP: Mont. Const. Art. X, sec. 9, 20-2-121, 20-3-106, 20-7-101, MCA

10.53.507 MONTANA GRADE 5 MATHEMATICS CONTENT STANDARDS

(1) Mathematics operations and algebraic thinking content standards for Grade 5 are to:

(a) use parentheses, brackets, or braces in numerical expressions and evaluate expressions with these symbols using the order of operations;

(b) write simple expressions that record calculations with numbers and interpret numerical expressions without evaluating them; ~~for example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7); recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product;~~ and

(c) generate two numerical patterns using ~~two~~ given rules and complete an input-output table for the data; identify apparent relationships between corresponding terms; form ordered pairs ~~consisting of corresponding terms from the two patterns and graph the ordered pairs~~ from the values in the input-output table and graph them on a coordinate plane~~; for example, given the rule "add 3" and the starting number 0, and given the rule "add 6" and the starting number 0, generate terms in the resulting sequences and observe that the terms in one sequence are twice the corresponding terms in the other sequence; and explain informally why this is so~~.

(2) Mathematics number and operations in base ten content standards for Grade 5 are to:

(a) recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left;

(b) explain patterns in the number of zeros of the product when multiplying a number by powers of 10; explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10; and use whole-number exponents to denote powers of 10;

(c) read, write, and compare decimals to thousandths~~;~~ by:

(i) ~~read~~ reading and ~~write~~ writing decimals to thousandths using ~~base ten numerals, number names,~~ standard form, word form, and expanded form~~, e.g. 347.392 = 3 × 100 +4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000)~~; and

(ii) ~~compare~~ comparing two decimals to thousandths based on meanings of the digits in each place using >, =, and < symbols ~~to record the results of comparisons~~;

(d) use place value understandings to round decimals to any place;

(e) ~~fluently~~ accurately and efficiently multiply multi-digit whole numbers using the standard algorithm;

(f) flexibly, accurately, and efficiently find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division and illustrate and explain the calculation by using equations, rectangular arrays, and/or area models; and

(g) add, subtract, multiply, and divide decimals to hundredths using concrete models or drawings ~~within cultural contexts, including those of Montana American Indians, and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method; and explain the reasoning used~~. This standard should incorporate designs and cultural context relating to Montana Indigenous Peoples and local communities.

(3) Mathematics number and operations - fractions content standards for Grade 5 are to:

(a) add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions ~~in such a way as to produce an equivalent sum or difference of fractions with like denominators; for example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12 (in general, a/b + c/d = (ad + bc)/bd)~~;

(b) solve ~~word~~ problems in context that involve ~~involving~~ addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, ~~e.g., by using visual fraction models or equations to represent the problem;~~ and use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers; ~~for example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2;~~

(c) interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b)~~;~~ and solve ~~word~~ problems in context that involve ~~involving~~ division of whole numbers leading to answers in the form of fractions or mixed numbers~~, e.g., by using visual fraction models or equations to represent the problem; for example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3 and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4; if 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?~~;

(d) apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction~~;~~ by:

(i) ~~interpret~~ expressing the product (a/b) × q as "a" parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b; ~~for example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation within cultural contexts, including those of Montana American Indians; and do the same with (2/3) × (4/5) = 8/15 (in general, (a/b) × (c/d) = ac/bd);~~ and

(ii) ~~find~~ finding the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths; ~~show~~ showing that the area is the same as would be found by multiplying the side lengths; ~~multiply~~

multiplying fractional side lengths to find areas of rectangles; and ~~represent~~ representing fraction products as rectangular areas;

(e) interpret multiplication as scaling (resizing), by:

(i) comparing the size of a product to the size of one factor on the basis of the size of the other factor without performing the indicated multiplication; and

(ii) explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number ~~(recognizing multiplication by whole numbers greater than 1 as a familiar case)~~; explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (n × a)/(n × b) to the effect of multiplying a/b by 1;

(f) solve ~~real-world~~ problems in context that involve ~~involving~~ multiplication of fractions and mixed numbers~~, e.g., by using visual fraction models or equations to represent the problem within cultural contexts, including those of Montana American Indians~~. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities;

(g) apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions~~;~~ by:

(i) ~~interpret~~ expressing division of a unit fraction by a nonzero whole number and compute such quotients; ~~for example, create a story context within cultural contexts, including those of Montana American Indians, for (1/3) ÷ 4, and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3;~~

(ii) ~~interpret~~ expressing division of a whole number by a unit fraction and compute such quotients; ~~for example, create a story context within cultural contexts, including those of Montana American Indians, for 4 ÷ (1/5), and use a visual fraction model to show the quotient; and use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4;~~ and

(iii) ~~solve~~ solving ~~real-world~~ problems in context involving division of unit fractions by nonzero whole numbers and division of whole numbers by unit fractions~~, e.g., by using visual fraction models and equations to represent the problem; for example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?~~. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities.

(4) Mathematics measurement and data content standards for Grade 5 are to:

(a) convert among different-sized standard measurement units within a given measurement system ~~(e.g., convert 5 cm to 0.05 m)~~ and use these conversions in solving multi-step~~, real-world~~ problems ~~within a cultural context, including those of Montana American Indians;~~ in context. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities;

(b) make a line plot to display a data set of measurements in fractions of a unit ~~(1/2, 1/4, 1/8);~~ and use operations on fractions ~~for this grade~~ to solve problems involving information presented in line plots; ~~for example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally;~~

(c) recognize volume as an attribute of solid figures and understand concepts of volume measurement~~;~~ by:

(i) understanding that a cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume and can be used to measure volume; and

(ii) understanding that a solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units;

(d) measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and ~~improvised~~ non-standard units;

(e) relate volume to the operations of multiplication and addition and ~~solve real-world and mathematical problems involving volume within cultural contexts, including those of Montana American Indians;~~ volume problems including problems in context by:

(i) ~~find~~ finding the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes and ~~show~~ showing that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base~~;~~ and ~~represent~~ representing ~~threefold whole-number products as volumes, e.g., to represent~~ the product of three whole numbers using the associative property of multiplication;

(ii) ~~apply~~ applying the formulas V = l × w × h and V = ~~b~~B × h for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths~~,~~ ~~in the context of solving real-world and mathematical problems~~ including problems in context; and

(iii) ~~recognize~~ recognizing volume as additive and ~~find~~ finding volumes of solid figures composed of two nonoverlapping right rectangular prisms by adding the volumes of the nonoverlapping parts, applying this technique to solve ~~real-world~~ problems in context. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities.

(5) Mathematics geometry content standards for Grade 5 are to:

(a) use a pair of perpendicular number lines, called axes, to define a coordinate system with the intersection of the lines ~~(~~at the origin~~)~~ arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates; understand that the x-coordinate, the first number, indicates how far to travel from the origin in the direction of ~~one~~ the x-axis and the y-coordinate, the second number, indicates how far to travel in the direction of the ~~second~~ y-axis~~, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate)~~;

(b) represent ~~real-world and mathematical~~ problems including problems in context by graphing points in the first quadrant of the coordinate plane and interpret coordinate values of points in the context of the situation~~, including those found in Montana American Indian designs~~. This standard should incorporate designs and cultural context relating to Montana Indigenous Peoples and local communities;

(c) understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category; ~~for example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles;~~ and

(d) classify two-dimensional figures in a hierarchy based on properties.

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IMP: Mont. Const. Art. X, sec. 9, 20-2-121, 20-3-106, 20-7-101, MCA

10.53.508 MONTANA GRADE 6 MATHEMATICS CONTENT STANDARDS

(1) Mathematics ratios and proportional relationship content standards for Grade 6 are to:

(a) understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities; ~~for example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."~~

(b) understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship; and ~~for example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."~~

(c) use ratio and rate reasoning to solve ~~real-world and mathematical~~ proportional problems ~~from a variety of cultural contexts, including those of Montana American Indians, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations;~~ in context about unit rates, percentages (as a rate per 100), and/or measurement units using tables or equations. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities.

~~(i) make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, plot the pairs of values on the coordinate plane, and use tables to compare ratios;~~

~~(ii) solve unit rate problems including those involving unit pricing and constant speed; for example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? As a contemporary American Indian example, it takes at least 16 hours to bead a Crow floral design on moccasins for two children. How many pairs of moccasins can be completed in 72 hours?;~~

~~(iii) find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity) and solve problems involving finding the whole, given a part and the percent;~~

~~(iv) use ratio reasoning to convert measurement units and manipulate and transform units appropriately when multiplying or dividing quantities.~~

(2) Mathematics number system content standards for Grade 6 are to:

(a) represent, interpret, and compute quotients of fractions and solve ~~word~~ problems in context involving division of fractions by fractions~~, e.g., by using visual fraction models and equations to represent the problem; for example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?~~;

(b) ~~fluently~~ accurately and efficiently divide multi-digit numbers using the standard algorithm;

(c) ~~fluently~~ accurately and efficiently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation;

(d) find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12; use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor; ~~for example, express 36 + 8 as 4 (9 + 2);~~

(e) understand that positive and negative numbers are used together to describe quantities having opposite directions or values ~~(e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge)~~ and use positive and negative numbers to represent quantities in ~~real-world~~ problems in context~~s~~, explaining the meaning of 0 in each situation;

(f) understand a rational number as a point on the number line and extend number line diagrams and coordinate axes ~~familiar from previous grades to represent points on the line and in the plane with negative number coordinates;~~ by:

(i) ~~recognize~~ recognizing opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; ~~recognize~~ recognizing that the opposite of the opposite of a number is the number itself, ~~e.g., - (-3) = 3;~~ and that 0 is its own opposite;

(ii) ~~understand~~ understanding signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane and ~~recognize~~ recognizing that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes; and

(iii) ~~find~~ finding and ~~position~~ positioning integers and other rational numbers on a horizontal or vertical number line diagram and ~~find~~ finding and ~~position~~ positioning pairs of integers and other rational numbers on a coordinate plane;

(g) understand ordering and absolute value of rational numbers~~;~~ by:

(i) ~~interpret~~ interpreting statements of inequality as statements about the relative position of two numbers on a number line diagram; ~~for example, interpret -3 > -7 as a statement that -3 is located to the right of -7 on a number line oriented from left to right;~~

(ii) ~~write~~ writing, ~~interpret~~ interpreting, and ~~explain~~ explaining statements of order for rational numbers in ~~real-world contexts; for example, write -3°C > -7°C to express the fact that -3°C is warmer than -7°C~~ problems in context;

(iii) ~~understand~~ understanding the absolute value of a rational number as its distance from 0 on the number line~~;~~ and ~~interpret~~ interpreting absolute value as magnitude for a positive or negative quantity in ~~a real-world situation; for example, for an account balance of -30 dollars, write |-30| = 30 to describe the size of the debt in dollars~~ problems in context; and

(iv) ~~distinguish~~ distinguishing comparisons of absolute value from statements about order; ~~for example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars;~~

(h) ~~solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, by graphing~~ graph points in all four quadrants of the coordinate plane and include the use of coordinates and

absolute value to find distances between points with the same first coordinate or the same second coordinate. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities.

(3) Mathematics expressions and equations content standards for Grade 6 are to:

(a) write and evaluate numerical expressions involving whole-number bases and exponents;

(b) write, read, and evaluate expressions ~~in which letters stand for numbers;~~ with variables by:

(i) ~~write~~ writing expressions that record operations with numbers and with ~~letters standing for numbers; for example, express the calculation "subtract y from 5" as 5 - y~~ variables;

(ii) ~~identify~~ identifying parts of an expression using mathematical terms ~~(sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity; for example, describe the expression 2 (8 + 7) as a product of two factors; and view (8 + 7) as both a single entity and a sum of two terms; and~~ (sum, product, difference, quotient, term, factor, coefficient, variable) and writing expressions that represent verbal descriptions of problems in context; and

(iii) ~~evaluate~~ evaluating expressions at specific values of their variables~~;~~ ~~include~~ including expressions that arise from formulas ~~used in real-world problems;~~, ~~perform~~ performing arithmetic operations, including those involving whole-number exponents ~~in the conventional order when there are no parentheses to specify a particular order (~~, and using the order of operations~~); for example, use the formulas V = s3 and A = 6 s2 to find the volume and surface area of a cube with sides of length s = 1/2~~;

(c) apply the properties of operations including the distributive property, to generate equivalent expressions~~; for example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); and apply properties of operations to y + y + y to produce the equivalent expression 3y;~~ and determine when two expressions are equivalent;

~~(d) identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them); for example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for;~~

~~(e)~~(d) understand ~~solving~~ how to solve an equation or inequality as a process ~~of answering a question: which values from a specified set, if any, make the equation or inequality true? Use~~ by using substitution to determine whether a given number in a specified set makes an equation or inequality true;

~~(f)~~(e) ~~use variables to represent numbers and~~ write expressions when solving ~~a real-world or mathematical~~ ~~problem~~ problems in context and understand that a variable can represent an unknown number, or~~, depending on the purpose at hand,~~ any number in a specified set;

~~(g)~~(f) solve ~~real-world and mathematical~~ problems including problems in context by writing and solving equations of the form x + p = q and px = q for cases in which p, q, and x are all nonnegative rational numbers;

~~(h)~~(g) write an inequality of the form x > c or x < c to represent a constraint or condition in ~~a real-world or mathematical problem; recognize that inequalities of the form x > c or x < c have infinitely many solutions; and represent~~ problems including problems in context; graph and describe solutions of such inequalities on number line diagrams; and

~~(i)~~(h) use variables to represent two quantities ~~in a real-world problem from a variety of cultural contexts, including those of Montana American Indians,~~ that change in relationship to one another~~; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable;~~ analyze the relationship between the dependent and independent variables using graphs and tables, ~~and relate these to the equation; for example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times and write the equation d = 65t to represent the relationship between distance and time.~~ and write an equation to express one quantity in terms of the other; this standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities.

(4) Mathematics geometry content standards for Grade 6 are to:

(a) find the area of ~~right~~ triangles, ~~other triangles, special~~ quadrilaterals, and other polygons by composing them into rectangles or decomposing them into triangles and other shapes; ~~apply these techniques in the context of solving real-world and mathematical problems within cultural contexts, including those of Montana American Indians; for example, use Montana American Indian designs to decompose shapes and find the area~~. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities;

(b) find the volume of a right rectangular prism with fractional edge lengths by ~~packing~~ filling it with unit cubes of the appropriate unit fraction edge lengths ~~and show that the volume is the same as would be found by multiplying the edge lengths of the prism~~ and connect and apply the formulas V = l w h and V = ~~b~~B h to find volumes of right rectangular prisms with fractional edge lengths ~~in the context of solving real-world and mathematical problems~~ to solve problems in context;

(c) draw polygons in the coordinate plane given coordinates for the vertices~~;~~ ~~use coordinates to~~, find the length of a ~~side joining points with the same first coordinate or the same second coordinate;~~ horizontal or vertical side and apply these techniques ~~in the context of solving real-world and mathematical~~ to problems in context; and

(d) represent three-dimensional figures using nets made up of rectangles and triangles and use the nets to find the surface area of these figures ~~and apply these techniques in the context of solving real-world and mathematical problems within cultural contexts, including those of Montana American Indians.~~ in problems including problems in context. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities.

(5) Mathematics statistics and probability content standards for Grade 6 are to:

(a) recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers; ~~for example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages;~~

(b) understand that a set of data collected ~~(including Montana American Indian demographic data)~~ to answer a statistical question has a distribution ~~which~~ that can be described by its center, spread, and overall shape. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities;

(c) recognize that ~~a measure~~ measures of ~~center~~ central tendency for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number;

(d) display numerical data in plots on a number line, including dot plots, histograms, and box plots and describe any overall pattern and any striking deviations from the overall pattern with reference to the context which the data were gathered; and

(e) ~~summarize~~ characterize numerical data sets from a sample in relation to their context, such as by:

(i) reporting the number of observations;

(ii) describing the nature of the attribute under investigation, including how it was measured and its units of measurement; and

(iii) ~~giving~~ finding quantitative measures of ~~center~~ central tendency (mode, median, and/or mean) and variability (interquartile range and/or mean absolute deviation), ~~as well as describing any overall pattern and any striking deviations from the overall pattern with reference to~~ for numerical data sets and relating the choice of measures of central tendency and variability to the shape of the data distribution and the context in which the data were gathered. ~~; and~~

~~(iv) relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.~~

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10.53.509 MONTANA GRADE 7 MATHEMATICS CONTENT STANDARDS

(1) Mathematics ratios and proportional relationship content standards for Grade 7 are to:

(a) compute unit rates associated with ratios of fractions, ~~including ratios of lengths, areas and other quantities~~ measured in like or different units; ~~for example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2 / 1/4 miles per hour, equivalently 2 miles per hour;~~

(b) recognize and represent proportional relationships between quantities, ~~including those represented in Montana American Indian cultural contexts;~~ using tables, graphs, and equations by:

(i) ~~decide~~ deciding whether ~~two~~ a table represents quantities ~~are~~ in a proportional relationship, ~~e.g.,~~ by testing for equivalent ratios ~~in a table or graphing on a coordinate plane and observing~~ and deciding whether a graph represents quantities in a proportional relationship if the graph is a straight line through the origin; and

(ii) ~~identify~~ identifying the constant of proportionality (unit rate) in tables, graphs, and equations, ~~diagrams, and verbal descriptions~~ of proportional relationships;

~~(iii) represent proportional relationships by equations; for example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn; as a contemporary American Indian example, analyze cost of beading materials; cost of cooking ingredients for family gatherings, community celebrations, etc.; and~~

~~(iv) explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate;~~

(c) use proportional relationships to solve multi-step ratio and percent problems, ~~within cultural contexts, including those of Montana American Indians (e.g., percent of increase and decrease of tribal land); for example:~~ including problems in context that involve simple interest, tax, markups and markdowns, gratuities and commissions, fees, and percent increase and decrease, percent error. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities.

(2) Mathematics number system content standards for Grade 7 are to:

(a) ~~apply and extend previous understandings of addition and subtraction to~~ add and subtract rational numbers, ~~and~~ represent addition and subtraction on a horizontal or vertical number line diagram, and understand subtraction as adding the additive inverse p - q = p + (-q);

~~(i) describe situations in which opposite quantities combine to make 0; for example, a hydrogen atom has 0 charge because its two constituents are oppositely charged;~~

~~(ii) understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is positive or negative; show that a number and its opposite have a sum of 0 (are additive inverses); and interpret sums of rational numbers by describing real-world contexts;~~

~~(iii) understand subtraction of rational numbers as adding the additive inverse, p – q = p + (–q); show that the distance between two rational numbers on the number line is the absolute value of their difference; and apply this principle in real-world contexts; and~~

~~(iv) apply properties of operations as strategies to add and subtract rational numbers;~~

(b) ~~apply and extend previous understandings of multiplication and division and of fractions to~~ multiply and divide rational numbers~~;~~ and use operations of rational numbers to solve problems in context. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities; and

~~(i) understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers; and interpret products of rational numbers by describing real-world contexts;~~

~~(ii) understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with nonzero divisor) is a rational number, i.e. if p and q are integers, then –(p/q) = (–p)/q = p/(–q); and interpret quotients of rational numbers by describing real-world contexts;~~

~~(iii) apply properties of operations as strategies to multiply and divide rational numbers; and~~

~~(iv)~~(c) ~~convert a~~ write any rational number ~~to~~ as a fraction, decimal, and percent using long division~~;~~ and know that the decimal form of a rational number terminates ~~in 0s~~ or ~~eventually~~ repeats~~;~~.

~~(c) solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, involving the four operations with rational numbers.~~

(3) Mathematics expressions and equations content standards for Grade 7 are to:

(a) ~~apply~~ use properties of operations ~~as strategies~~ to add, subtract, factor, and expand linear expressions with rational coefficients and generate equivalent expressions;

(b) understand that rewriting an expression in different forms in a problem in context can ~~shed light on the problem and~~ show how the quantities in it are related; ~~for example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05;"~~

(c) write and solve ~~multistep real-life and mathematical~~ one- and two-step equations including problems ~~posed with positive and negative~~ in context with rational numbers, ~~in any form (whole numbers, fractions, and decimals), using tools strategically; apply properties of operations to calculate with numbers in any form;~~ convert between forms as appropriate~~;~~, and assess the reasonableness of answers ~~using mental computation and estimation strategies; for example: if a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50 and if you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation~~; and

(d) use variables to represent quantities ~~in a real-world or mathematical problems, including those represented in Montana American Indian cultural contexts,~~ and construct simple equations and inequalities to solve problems ~~by reasoning about the quantities;~~ in context. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities by:

(i) ~~solve word~~ solving, accurately and efficiently, problems in context leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers~~;~~, ~~solve equations of these forms fluently; compare~~ comparing an algebraic solution to an arithmetic solution, and identifying the sequence of the operations used in each approach; ~~for example, the perimeter of a rectangle is 54 cm. and its length is 6 cm. What is its width?;~~ and

(ii) ~~solve word~~ solving problems in context leading to inequalities of the form px + q > r or px + q < r, where p, q, and r are specific rational numbers~~; graph~~, graphing the solution set of the inequality, and ~~interpret~~ interpreting ~~it in the context of the problem;~~ the solution in context. ~~for example: as a salesperson, you are paid $50 per week plus $3 per sale; this week you want your pay to be at least $100; write an inequality for the number of sales you need to make and describe the solutions.~~

(4) Mathematics geometry content standards for Grade 7 are to:

(a) solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale;

(b) draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions; focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle;

~~(c) describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids;~~

~~(d)~~(c) know and use the formulas for the area and circumference of a circle ~~and use them to solve problems from a variety of cultural contexts, including those of Montana American Indians~~ and give an informal derivation of the relationship between the circumference and area of a circle. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities;

~~(e)~~(d) use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure; and

~~(f)~~(e) solve ~~real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians,~~ geometrical problems including problems in context that involve ~~involving~~ area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities.

(5) Mathematics statistics and probability content standards for Grade 7 are to:

(a) understand ~~that~~ statistics can be used to gain information about a population by examining a representative sample of the population; ~~generalizations about a population from a sample are valid only if the sample is representative of that population; and understand that random sampling tends to produce representative samples and support valid inferences;~~

(b) use data, ~~including Montana American Indian demographics data,~~ from a random sample to draw inferences about a population with an unknown characteristic of interest~~;~~ and generate or simulate multiple samples ~~(or simulated samples)~~ of the same size to gauge the variation in estimates or predictions; ~~for example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data; predict how many text messages your classmates receive in a day and gauge how far off the estimate or prediction might be~~. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities;

(c) ~~informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability; for example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is~~

~~noticeable~~ visually analyze two data distributions to compare measures of central tendency and variability;

(d) use measures of ~~center~~ central tendency and measures of variability for numerical data from random samples to draw ~~informal~~ comparative inferences about two populations; ~~for example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book;~~

(e) understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring; ~~larger numbers indicate greater likelihood; a probability near 0 indicates an unlikely event; a probability around 1/2 indicates an event that is neither unlikely nor likely; and a probability near 1 indicates a likely event;~~

(f) ~~approximate the~~ find the experimental probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency. ~~and predict the approximate relative frequency given the probability; for example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times and when playing Montana American Indian hand/stick games, you can predict the approximate number of accurate guesses~~ This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities;

(g) develop a theoretical probability model and use it to find probabilities of events~~;~~, compare theoretical and experimental probabilities, ~~from a model to observed frequencies; and if the agreement is not good,~~ and explain possible sources of ~~the~~ discrepancy, if any exist; and

~~(i) develop a uniform probability model by assigning equal probability to all outcomes and use the model to determine probabilities of events; for example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected; and~~

~~(ii) develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process; for example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down; do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?;~~

(h) represent sample spaces for compound events, identify the desired outcomes in the sample spaces, and find probabilities of ~~compound~~ events using organized lists, tables, tree diagrams, and ~~simulation;~~ simulations.

~~(i) understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs;~~

~~(ii) represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams; for an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event; and~~

~~(iii) design and use a simulation to generate frequencies for compound events; for example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?.~~

AUTH: Mont. Const. Art. X, sec. 9, 20-2-114, 20-7-101, MCA
IMP: Mont. Const. Art. X, sec. 9, 20-2-121, 20-3-106, 20-7-101, MCA

10.53.510 MONTANA GRADE 8 MATHEMATICS CONTENT STANDARDS

(1) Mathematics number system content standards for Grade 8 are to:

(a) know real numbers are made up of rational and irrational numbers, understand informally that every number has a decimal expansion~~; for rational numbers show that the decimal expansion repeats eventually;~~ and convert a decimal expansion which repeats eventually into a rational number; and

(b) use rational approximations of irrational numbers to compare the ~~size~~ value of irrational numbers~~;~~, locate them approximately on a number line diagram~~;~~, and estimate the value of expressions ~~(e.g., π2); for example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations~~.

(2) Mathematics expressions and equations content standards for Grade 8 are to:

(a) know and apply the properties of integer exponents to generate equivalent numerical expressions; ~~for example, 3^2 ¡Á $3^{¨C5}$ = $3^{¨C3}$ = $1/3^3$ = 1/27;~~

(b) use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number; evaluate square roots of small perfect squares and cube roots of small perfect cubes; ~~and know that ¡Ì2 is irrational;~~

(c) ~~use numbers expressed in the form of a single digit times a whole-number power of 10 to estimate~~ represent very large or very small quantities~~,~~ ~~and to express how many times as much one is than the other; for example, estimate the population of the United States as 3 times 108 and the population of the world as 7 times 109 and determine that the world population is more than 20 times larger;~~ using scientific notation, limited to a single digit times an integer power of ten;

(d) perform operations with numbers expressed in scientific notation~~,~~ ~~including problems where both decimal and scientific notation are used; use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading); and interpret scientific notation that has been generated by technology~~;

(e) graph proportional relationships, interpreting the unit rate as the slope of the graph~~;~~ and compare two different proportional relationships ~~represented in different ways; for example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed~~ as tables, graphs, and equations;

(f) use similar triangles to explain why the slope m is the same between any two distinct points on a nonvertical line in the coordinate plane~~;~~ and derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b;

(g) solve linear equations in one variable~~;~~ by:

(i) ~~give~~ giving examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions and ~~show~~ showing which of these possibilities is the case by successively transforming the given equation into simpler

forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers); and

(ii) ~~solve~~ solving linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms; and

(h) analyze and solve pairs of simultaneous linear equations~~;~~ by:

(i) ~~understand~~ understanding that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously;

(ii) ~~solve~~ solving systems of two linear equations in two variables algebraically ~~and estimate~~, estimating solutions by graphing the equations~~; solve~~, and solving simple cases by inspection; ~~for example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6;~~ and

(iii) ~~solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, leading~~ solving problems in context that lead to two linear equations in two variables; ~~for example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair~~. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities.

(3) Mathematics functions content standards for Grade 8 are to:

(a) understand that a function is a rule that assigns to each input exactly one output and the graph of a function is the set of ordered pairs (x,y) each consisting of an input, x, and the corresponding output, y;

(b) compare properties of two functions ~~each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions); for example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change~~ using tables, graphs, and equations;

(c) interpret the equation y = mx + b as defining a linear function whose graph is a straight line~~; give examples of functions that are not linear; for example, the function A = s2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4), and (3,9), which are not on a straight line~~ with slope m passing through the point (0, b);

(d) given linear data relating two quantities, construct a linear function ~~to model a linear relationship between two quantities; determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph;~~ that models the data; and interpret the rate of change and initial value of a linear function in terms of the situation it models ~~and in terms of its graph or a table of values~~; and

(e) given the graph of a function, describe qualitatively the functional relationship between ~~two~~ quantities ~~by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear)~~ and given a verbal description of a functional relationship, sketch a graph that exhibits the qualitative features of a function ~~that has been described verbally~~.

(4) Mathematics geometry content standards for Grade 8 are to:

(a) verify experimentally the properties of rotations, reflections, and translations ~~from a variety of cultural contexts, including those of Montana American Indians:~~ and understand that these are rigid transformations, lines are taken to lines, line segments to line segments of the same length, angles are taken to angles of the same measure, and parallel lines are taken to parallel lines. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities;

~~(i) lines are taken to lines and line segments to line segments of the same length;~~

~~(ii) angles are taken to angles of the same measure; and~~

~~(iii) parallel lines are taken to parallel lines;~~

(b) understand that a two-dimensional figure is congruent to another if the second can be obtained ~~from the first by a sequence of rotations, reflections, and translations~~ by a sequence of rigid transformations and given two congruent figures, describe a sequence that exhibits the congruence between them;

(c) describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures ~~from a variety of cultural contexts, including those of Montana American Indians,~~ using coordinates. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities;

(d) understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations and given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them;

(e) use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles; ~~for example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line and give an argument in terms of transversals why this is so;~~

(f) explain a proof of the Pythagorean Theorem and its converse;

(g) apply the Pythagorean Theorem to determine unknown side lengths in right triangle~~s in real-world and mathematical~~ problems, including problems in context in two and three dimensions; ~~for example, determine the unknown height of a Plains Indian tipi when given the side length and radius~~. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities;

(h) apply the Pythagorean Theorem to find the distance between two points in a coordinate system; and

(i) know, use, and apply the formulas for the volumes of cones, cylinders, and spheres ~~and use them~~ to solve ~~real-world and mathematical~~ problems, including problems in context.

(5) Mathematics statistics and probability content standards for Grade 8 are to:

(a) construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities and describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association;

(b) know that straight lines are widely used to model relationships between two quantitative variables and for scatter plots that suggest a linear association, informally fit a straight line and informally assess the model fit by judging the closeness of the data points to the line;

(c) use the equation of a linear model to solve problems in the context of bivariate measurement data~~, interpreting~~ and interpret the slope and intercept; ~~for example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height;~~

(d) ~~understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table; construct and interpret a two-way table summarizing data including data from Montana American Indian sources on two categorical variables collected from the same subjects; use relative frequencies calculated for rows or columns to describe possible association between the two variables; for example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?~~ construct and interpret frequencies and relative frequencies for bivariate categorical data in a two-way table to investigate patterns of association. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities.

AUTH: Mont. Const. Art. X, sec. 9, 20-2-114, 20-7-101, MCA
IMP: Mont. Const. Art. X, sec. 9, 20-2-121, 20-3-106, 20-7-101, MCA

10.53.511 ~~SYMBOLS~~ DEFINITIONS (1) ~~The symbol "+" denotes science, technology, engineering, mathematics (STEM) standards that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics.~~ The Montana High School Math Standards are broken into two groups, Core and Core Plus. Together these standards cover all essential concepts for high school. These terms are defined in the following manner:

~~(2)~~(a) ~~The symbol "*" denotes specific modeling standards appearing throughout the high school mathematics standards.~~ Core standards: foundational standards that all Montana students should know, understand, and be able to do upon graduation of high school; and

(b) Core Plus: additional standards that all Montana students can pursue to prepare for postsecondary education and careers.

AUTH: Mont. Const. Art. X, sec. 9, 20-2-114, 20-7-101, MCA
IMP: Mont. Const. Art. X, sec. 9, 20-2-121, 20-3-106, 20-7-101, MCA

5. The rules proposed to be repealed are as follows:

10.53.512 MONTANA HIGH SCHOOL MATHEMATICS NUMBER AND QUANTITY STANDARDS

AUTH: 20-2-114, MCA

IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.53.513 MONTANA HIGH SCHOOL MATHEMATICS ALGEBRA CONTENT STANDARDS

AUTH: 20-2-114, MCA
IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.53.514 MONTANA HIGH SCHOOL MATHEMATICS FUNCTIONS STANDARDS

AUTH: 20-2-114, MCA
IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.53.515 MONTANA HIGH SCHOOL MATHEMATICS MODELING CONTENT STANDARDS

AUTH: 20-2-114, MCA
IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.53.516 MONTANA HIGH SCHOOL MATHEMATICS GEOMETRY CONTENT STANDARDS

AUTH: 20-2-114, MCA
IMP: 20-2-121, 20-3-106, 20-7-101, MCA

10.53.517 MONTANA HIGH SCHOOL MATHEMATICS STATISTICS AND PROBABILITY STANDARDS

AUTH: 20-2-114, MCA
IMP: 20-2-121, 20-3-106, 20-7-101, MCA

REASON: By authority of 20-7-101, MCA, standards of accreditation for all schools are adopted by the board upon the recommendation of the Superintendent of Public Instruction. The board considers recommendations for revision of the policies at any time it deems necessary and conducts a comprehensive review of standards of accreditation policies on a regular cycle to ensure that such policies are meeting the needs of the state. There have been numerous revisions over the last decade, but the last comprehensive review for math standards (rules) was in 2011.

As a result, in 2023, under the direction of the Superintendent of Public Instruction, the Office of Public Instruction (OPI) led an appointed task force over five months and a Negotiated Rulemaking Committee (NRC) for an additional four months to meticulously review and refine the Montana Mathematics Content Standards. Both groups made intentional decisions to significantly reduce the number of instances where specific examples appear in the rules; this is not to say that they believe them unnecessary but would appear in guidance documents to free

up instruction for educators rather than dictate a list of specific examples that must be utilized. Many of the standards were revised with the intent to reduce the use of unnecessary language and to provide clarity to the concept contained therein. Often, revisions did not change the standard's intent but did shorten its length and utilized language more familiar to families and educators. It was determined that consistency in wording in application and expectation should occur within the rules, where standards indicate the specific necessity of application (though not limited to these standards alone), the phrase "problems in context" has replaced inconsistent language, such as "word problems," "real-world problems," or other variations. The task force and NRC made efforts to include the word "by:" at the end of a standard that contained substandard as well as change the subsequent verbs to present progressive verbs ("ing") in compliance.

These revisions incorporate the latest research on math teaching and learning, emphasizing critical concepts and mathematical practices, the genuine integration of the cultural heritage of Montana's American Indians, the improvement of early numeracy and number sense at the K-5 level, the inclusion of critical data science and financial literacy content throughout the K-12 standards, as well as the creation of diverse pathways for high school learners. The rules were also updated to reflect the developmental trajectory across different concept domains through grade levels, with a deliberate focus on refining standards for better clarity and comprehension, ensuring accessibility for all individuals. Expansion of fluency language in previous standards used the broad word "fluently" which was vague and difficult to quantify. These standards now use variations of "flexibly," "accurately," and/or "efficiently" where appropriate to provide more clarity regarding the specific way students can demonstrate fluency.

The mathematical practices were amended to demonstrate recent research, including a notable innovative approach to incorporate cultural connections within the Mathematical Practice Standards; this approach stands out as the first of its kind nationwide. Supported by research advocating for students to relate math concepts to their own world and community, these proposed rules align with the Montana constitutional commitment and statutes to authentically integrate Indian Education for All (IEFA) within the standards. Previously, the cultural connections, or IEFA statements, existed in the middle of individual standards. These statements now appear as their own clauses at the end of some standards. This placement provides more emphasis on the IEFA component and contributes to the increased clarity of the standard itself. The statements have also been updated to include culturally responsive language and expanded to emphasize local communities, highlighting the intention that these standards relate to the community and culture(s) of the Indigenous Tribal Nations that exist, or historically existed, in the geographical region in which they are taught.

These updated rules were written to consider ways secondary math pathways could better fit the college and career goals for Montana high school students. Due to this effort, the proposed high school standards for mathematics have undergone major structural adaptations. These standards have been proposed in two categories: "Core," which is defined as "foundational standards that all Montana students should know and be able to do upon graduation" and "Core Plus," which is defined as "additional standards that all Montana students can pursue for post-

secondary education and careers." The functions standards are split between Core and Core Plus as follows: standards related to linear, exponential, and quadratic functions are in the Core standards; standards related to polynomials, logarithmic functions, and trigonometric functions are in the Core Plus standards. The major improvement across the proposed standards is that modeling standards are now grouped together in dedicated sections in the algebra and geometry standards, rather than being distributed throughout the standards. In the time since the adoption of the previous standards, modeling has grown in importance. Having dedicated sections for modeling helps to underscore its importance, which modernizes the standards. The dedicated sections also improve the simplicity for teachers because they can see all modeling standards at once, rather than having to flip between many pages of standards. To improve simplicity, the proposed standards focus only on essential concepts related to number and quantity. In practice, this means that the proposed standards do not include standards related to matrices and vectors, nor advanced operations with complex numbers. In line with modern recommendations around data science, the proposed standards focus on reasoning with data in context, rather than abstract probability theory. In geometry, rather than treating transformations, constructions, and proof as discrete concepts and skills, the proposed standards unify them into a geometric reasoning process of exploring, conjecturing, and proving.

Regardless of a student's trajectory in math learning, the revised high school standards were designed to enable personalized learning, allowing students to progress at their own pace and demonstrate proficiency according to their individualized learning journeys. The standards are organized into domains and clusters to match common instructional units, providing flexibility for teachers to organize the standards into instructional units that fit their contexts. The new rules maintain mathematical rigor without using unnecessarily technical terms or overly wordy language, making them understandable to teachers, students, and families.

6. Concerned persons may submit their data, views, or arguments either orally or in writing at the hearing. Written data, views, or arguments may also be submitted to: McCall Flynn, Executive Director, 46 N. Last Chance Gulch, Suite 2B; P.O. Box 200601, Helena, MT 59620-0601; telephone (406) 444-0302; or email bpe@mt.gov, and must be received no later than 5:00 p.m., November 1, 2024.

7. McCall Flynn, executive director, has been designated to preside over and conduct this hearing.

8. The board maintains a list of interested persons who wish to receive notices of rulemaking actions proposed by this agency. Persons who wish to have their name added to the list shall make a written request that includes the name, email, and mailing address of the person to receive notices and specifies for which program the person wishes to receive notices. Notices will be sent by email unless a mailing preference is noted in the request. Such written request may be mailed or delivered to the contact person in paragraph 6 above or may be made by completing a request form at any rules hearing held by the board.

9. An electronic copy of this proposal notice is available through the Secretary of State's web site at rules.mt.gov.

10. The bill sponsor contact requirements of 2-4-302, MCA, do not apply.

11. With regard to the requirements of 2-4-111, MCA, the board has determined that the amendment of the above-referenced rules will not significantly and directly impact small businesses.

/s/ McCall Flynn	/s/ Tim Tharp
McCall Flynn	Tim Tharp
Rule Reviewer	Chair
Board of Public Education	Board of Public Education

Certified to the Secretary of State September 24, 2024.